

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	<u>55</u>

continuous

The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C , where $C(t)$ is measured in degrees Celsius. For $0 \leq t \leq 12$, selected values of $C(t)$ are given in the table shown.

- (c) For $12 \leq t \leq 20$, the rate of change of the temperature of the coffee is modeled by

$C'(t) = \frac{-24.55e^{0.01t}}{t}$, where $C'(t)$ is measured in degrees Celsius per minute. Find the temperature of the coffee at time $t = 20$. Show the setup for your calculations.

$$C(t) = \int_{12}^{20} \frac{-24.55e^{0.01T}}{T} dT = C(20) - C(12) \text{ change in TEMP}$$

\uparrow Temp at $T=20$ \uparrow TEMP $dT = R$
How much TEMP change

$$\int_{12}^{20} \frac{-24.55e^{0.01x}}{x} dx$$

went down

$= -14.670811944$

$$C(12) = 55$$

$$C(20) = 55 - 14.6708$$

$$= 55 - 14.670812 = 40.329188$$

For the model defined in part (c), it can be shown that $C''(t) = \frac{0.2455e^{0.01t}(100-t)}{t^2}$. For $T > 100$

$12 < t < 20$, determine whether the temperature of the coffee is changing at a decreasing rate or at an increasing rate. Give a reason for your answer.

$$\frac{C''(12)}{C''(20)} \text{ always + } \frac{0.2455e^{0.0(12)}(100-12)}{(12)^2}$$

$C''(t)$ tells us The Slope of Rate of Change of $C'(t)$
 $C''(t)$ is Rate of change For $C'(t)$
 $C'(t) = \text{Rate of Temp change}$

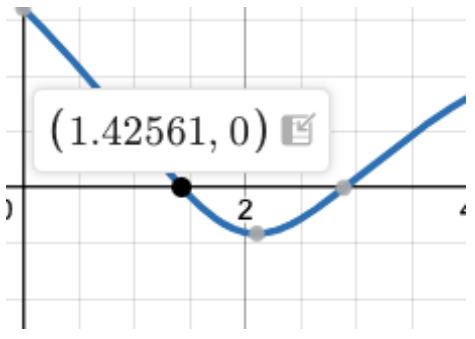
$C''(t) = - = \text{decreasing Rate}$
 $C''(t) = + = \text{increasing Rate}$

A particle moves along the x -axis so that its velocity at time $t \geq 0$ is given by $v(t) = \ln(t^2 - 4t + 5) - 0.2t$.

$$s(t) = \int v(t) dt$$

$$a(t) = v'(t) dt$$

There is one time, $t = t_R$, in the interval $0 < t < 2$ when the particle is at rest (not moving). Find t_R .
 For $0 < t < t_R$, is the particle moving to the right or to the left? Give a reason for your answer.



$t_R = 1.42561$
 $0 < t < 1.42561$
 $v(t) = + \text{ moving Right}$

Find the acceleration of the particle at time $t = 1.5$. Show the setup for your calculations. Is the speed of the particle increasing or decreasing at time $t = 1.5$? Explain your reasoning.

$$a(1.5) = -1$$

$$y = \frac{d}{dx} (\ln(x^2 - 4x + 5))$$

$$y = \frac{d}{dx} (\ln(x^2 - 4x + 5) - 0.2x) \quad \text{Trace To } x=1.5$$

$$v(1.5) = 0.07686$$

Speed is increasing $v(1.5) = -$
 $a(1.5) = -$

(c) The position of the particle at time t is $x(t)$, and its position at time $t = 1$ is $x(1) = -3$. Find the position of the particle at time $t = 4$. Show the setup for your calculations.

$$x(1) = -3 \quad \int_1^4 v(t) dt = \text{Change in Position from } T=1 \text{ to } T=4$$

$$y = \int_1^4 (\ln(x^2 - 4x + 5) - 0.2x) dx = 0.197116767815$$

Start at -3 when $T=1$

$$-3 + 0.19711677 = x(4) = -2.803$$

- (d) Find the total distance traveled by the particle over the interval $1 \leq t \leq 4$. Show the setup for your calculations.

$$\int_1^4 |v(t)| dt =$$

$$y = \int_1^4 \left| (\ln(x^2 - 4x + 5) - 0.2x) \right| dx$$

$$= 0.958133986858$$

↑
Total distance
Traveled

The depth of seawater at a location can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$, where $H(t)$ is measured in feet and t is measured in hours after noon ($t = 0$). It is known that $H(0) = 4$.

- (b) For $0 < t < 5$, it can be shown that $H(t) > 1$. Find the value of t , for $0 < t < 5$, at which H has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.

$\frac{dH}{dt} = 0$ or ϕ $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\frac{T}{2}$ (always +)
 $\cos\frac{T}{2} = 0$ or ϕ
 $\frac{T}{2} = \frac{\pi}{2}$ $T = \pi$ (critical #)
 Slope: + + + + + Max - - - - -
 0 1 2 3 π 4 5
 $\cos\frac{3}{2} = \cos 1 = +$ $\cos\frac{4}{2} = \cos 2 = -$
 $0 = \cos\frac{\pi}{2} = \cos\frac{3\pi}{2} = \cos -\frac{\pi}{2}$
 $(-)$ $(+)$ $(+)$ $(-)$
 $\frac{\pi}{2} = 1.57$
 (cos 0, sin 0)

(c) Use separation of variables to find $y = H(t)$, the particular solution to the differential equation

$$\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right) \text{ with initial condition } H(0) = 4.$$

$T=0 \quad H=4$

$$\frac{dH}{H-1} = \frac{1}{2}(H-1)\cos\left(\frac{T}{2}\right) \cdot dT$$

$\log_a b = c \Rightarrow a^c = b$
 $\ln_e a = b \Rightarrow e^b = a$

$$\int \frac{dH}{H-1} = \int \frac{1}{2} \cos \frac{T}{2} dT$$

$u = H-1$
 $du = dH$

$u = \frac{T}{2} = \frac{1}{2}T$
 $du = \frac{1}{2}dT$
 $2du = dT$

$$\int \frac{du}{u}$$

$$\int \frac{1}{2} \cos u \cdot 2 du$$

$$\int \cos u du = \sin u + C$$

$\ln|u|$

$\ln|H-1| = \sin \frac{T}{2} + C$

$\ln|H-1| = \sin \frac{T}{2} + C$

$\ln|4-1| = \sin \frac{0}{2} + C$

$\ln 3 = \sin 0 + C$

$\ln 3 = 0 + C$

$\ln 3 = C$

$\ln|H-1| = \sin \frac{T}{2} + \ln 3$

$e^{\sin \frac{T}{2} + \ln 3} = H-1$

$e^{\sin \frac{T}{2} + \ln 3} + 1 = H \rightarrow x^2 \cdot x^3 = x^{2+3}$

$e^{\sin \frac{T}{2}} \cdot e^{\ln 3} + 1 = H$

$3e^{\sin \frac{T}{2}} + 1 = H$

$\ln|H-1| + C_1 = \sin \frac{T}{2} + C_2$

$C_2 - C_1 = C_3$

$\ln|H-1| = \sin \frac{T}{2} + C_3$

Is the horizontal line $y = 1$ tangent to the curve? Give a reason for your answer.

$$m = 0 \quad \frac{dy}{dx} = 0 \quad \text{when } y = 1$$

Consider the curve defined by the equation $x^2 + 3y + 2y^2 = 48$. It can be shown that $\frac{dy}{dx} = \frac{-2x}{3+4y}$.

$$y = 1 \quad x = 0$$

$$0^2 + 3(1) + 2(1)^2 = 5 \neq 48$$

Point not
on curve

$$\frac{dy}{dx} = \frac{-2x}{3+4y} = \frac{-2x}{7}$$

$$\frac{dy}{dx} = 0 = \frac{-2x}{7}$$

$$x = 0$$

(c) The curve intersects the positive x -axis at the point $(\sqrt{48}, 0)$. Is the line tangent to the curve at this point vertical? Give a reason for your answer.

$$\frac{dy}{dx} = \phi$$

Nope

$$\frac{dy}{dx} = \frac{-2x}{3+4y} = \frac{-2\sqrt{48}}{3+4 \cdot 0} = \frac{-2\sqrt{48}}{3} \neq \phi$$

$$\frac{-2\sqrt{48}}{3} = \text{Slope of curve at } (\sqrt{48}, 0)$$

For time $t \geq 0$, a particle is moving along another curve defined by the equation $y^3 + 2xy = 24$. At the instant the particle is at the point $(4, 2)$, the y -coordinate of the particle's position is decreasing at a rate of 2 units per second. At that instant, what is the rate of change of the x -coordinate of the particle's position with respect to time?

$$y^3 + 2xy = 24$$

$$3y^2 \frac{dy}{dt} + 2 \frac{dx}{dt} \cdot y + 2x \frac{dy}{dt} = 0$$

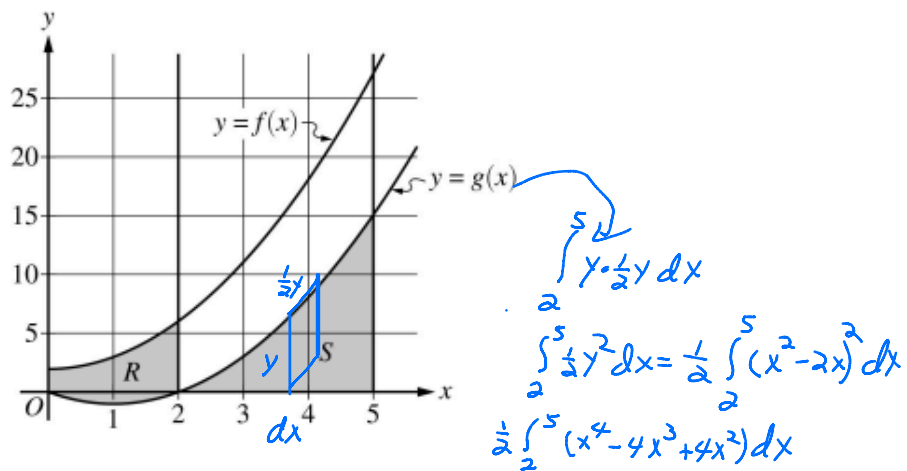
$$3(2)^2 \cdot -2 + 2 \cdot \frac{dx}{dt} \cdot 2 + 2(4) \cdot -2 = 0$$

$$\frac{dx}{dt}$$

$$-24 + 4 \frac{dx}{dt} - 16 = 0$$

$$4 \frac{dx}{dt} = 40$$

$$\frac{dx}{dt} = 10 \text{ u/sec}$$



The functions f and g are defined by $f(x) = x^2 + 2$ and $g(x) = x^2 - 2x$, as shown in the graph.

Model Solution

Scoring

- (a) Let R be the region bounded by the graphs of f and g , from $x = 0$ to $x = 2$, as shown in the graph. Write, but do not evaluate, an integral expression that gives the area of region R .

$$\int_0^2 (f(x) - g(x)) dx$$

-) Let S be the region bounded by the graph of g and the x -axis, from $x = 2$ to $x = 5$, as shown in the graph. Region S is the base of a solid. For this solid, at each x the cross section perpendicular to the x -axis is a rectangle with height equal to half its base in region S . Find the volume of the solid. Show the work that leads to your answer.

$$\frac{1}{2} \int_2^5 (x^4 - 4x^3 + 4x^2) dx = \frac{1}{2} \left[\frac{1}{5} x^5 - x^4 + \frac{4}{3} x^3 \right] \Big|_2^5$$

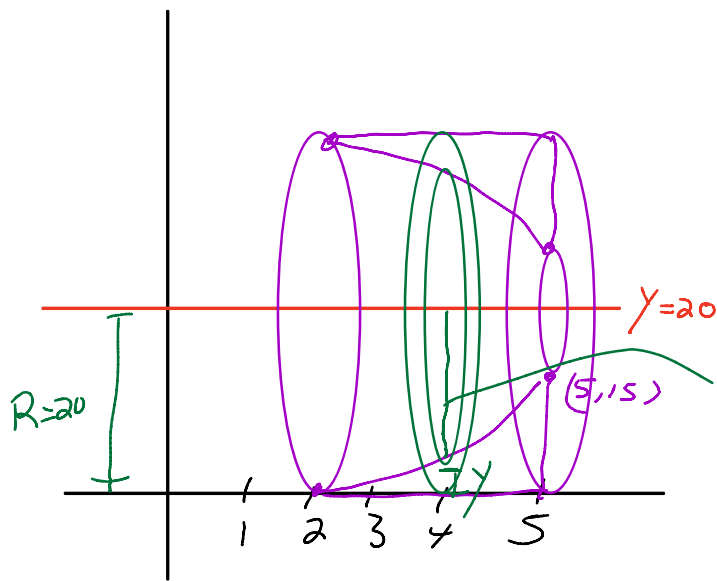
$$\frac{1}{2} \left[\frac{1}{5} (5)^5 - 5^4 + \frac{4}{3} (5)^3 \right] - \frac{1}{2} \left[\frac{1}{5} (2)^5 - 2^4 + \frac{4}{3} (2)^3 \right]$$

$$\frac{1}{2} \left[\cancel{5^4} - 5^4 + \frac{500}{3} \right] - \frac{1}{2} \left[\frac{32}{5} - 16 + \frac{24}{3} \right] = \frac{1}{2} \left[\frac{500}{3} - \frac{32}{5} + 16 - 8 \right]$$

$$\frac{500}{6} -$$

$$\frac{1}{2} \left[\frac{64}{5} - 16 + 8 \right]$$

$$\frac{500}{6} - \frac{32}{10} - 4 = \frac{414}{5}$$



$$r = 20 - y = 20 - g(x)$$

$$\pi \int_2^5 [R^2 - r^2] dx$$

$$\pi \int_2^5 [20^2 - (20 - (x^2 - 2x))^2] dx$$